Further Pure Mathematics FP3 (6669)

Mock paper mark scheme

Question number	Scheme	Marks
1.	$4\left(1-\text{sech}^2 x\right)-2\,\text{sech}^2 x=3$	M1
	$6\,\text{sech}^2 x=1$	A1
	$\cosh x=\sqrt{6}$	M1 A1
	Using $\text{arcosh}\, x=\ln\left(x+\sqrt{\left(x^2-1\right)}\right)$	M1
	$x=\pm\ln\left(\sqrt{6}+\sqrt{5}\right)$	A1 (6)
		(6 marks)
2.	$\dfrac{dy}{dx}=\sinh\left(\dfrac{1}{2}x\right)$	B1
	$\displaystyle\int y\sqrt{\left(1+\left(\frac{dy}{dx}\right)^2\right)}dx = \int 2\cosh\left(\frac{1}{2}x\right)\sqrt{\left(1+\sinh^2\left(\frac{1}{2}x\right)\right)}dx$	M1 A1
	$\displaystyle = \int 2\cosh\left(\frac{1}{2}x\right)dx$	
	$\displaystyle = \int\left(1+\cosh x\right)dx$	
	$= x+\sinh x$	M1 A1
	$\displaystyle 2\pi\left[x+\frac{e^x-e^{-x}}{2}\right]_{-\ln 2}^{\ln 2} = \pi\left[\left(2\ln 2+2-\frac{1}{2}\right)-\left(-2\ln 2+\frac{1}{2}-2\right)\right]$	M1
	$=\pi\left(4\ln 2+3\right)$	A1 (7)
		(7 marks)

Question number	Scheme	Marks
3.	$\dfrac{dx}{d\theta} = -\dfrac{3\cosh\theta}{\sinh^2\theta}$	B1
	$\displaystyle\int \frac{1}{x\sqrt{(x^2+9)}}dx = \int \frac{1}{\frac{3}{\sinh\theta}\sqrt{\left(\frac{9}{\sinh^2\theta}+9\right)}} \times \frac{-3\cosh\theta}{\sinh^2\theta}d\theta$	M1 A1
	$= -\dfrac{1}{3}\displaystyle\int 1 d\theta = -\frac{1}{3}\theta$	A1
	$x = 3\sqrt{3} \Rightarrow \sinh\theta = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = \ln\left(\dfrac{1}{\sqrt{3}} + \dfrac{2}{\sqrt{3}}\right) = \ln\sqrt{3}$	
	$x = 4 \Rightarrow \sinh\theta = \dfrac{3}{4} \Rightarrow \theta = \ln\left(\dfrac{3}{4} + \dfrac{5}{4}\right) = \ln 2$	M1 A1
	$\left[-\dfrac{1}{3}\theta\right]_{\ln 2}^{\ln\sqrt{3}} = \dfrac{1}{3}\left(\ln 2 - \ln\sqrt{3}\right) = \dfrac{1}{3}\left(\dfrac{1}{2}\ln 4 - \dfrac{1}{2}\ln 3\right) = \dfrac{1}{6}\ln\dfrac{4}{3}$	M1 A1 (8)
		(8 marks)
4. (a)	$\dfrac{dy}{dx} = \dfrac{1}{1+x} \times \dfrac{1}{2}x^{-\frac{1}{2}} \left(= \dfrac{1}{2x^{\frac{1}{2}}(1+x)} \right)$	M1 A1
	$x = \dfrac{1}{4} \Rightarrow \dfrac{dy}{dx} = \dfrac{4}{5}$	A1 (3)
(b)	$\dfrac{dy}{dx} = \dfrac{1}{2}(1+x)^{-1}x^{-\frac{1}{2}}$	
	$\dfrac{d^2y}{dx^2} = -\dfrac{1}{2}(1+x)^{-2} \times x^{-\frac{1}{2}} - \dfrac{1}{4}(1+x)^{-1} \times x^{-\frac{3}{2}}$	M1 A1
	$= -\dfrac{1+3x}{4x^{\frac{3}{2}}(1+x)^2}$	
	$2x(1+x)\dfrac{d^2y}{dx^2} + (1+3x)\dfrac{dy}{dx}$	
	$= 2x(1+x)\left(-\dfrac{1+3x}{4x^{\frac{3}{2}}(1+x)^2}\right) + (1+3x)\left(\dfrac{1}{2x^{\frac{1}{2}}(1+x)}\right)$	M1 A1, A1
	$= 0$ ✱	A1 cso (6)
		(9 marks)

Question number	Scheme	Marks
5. (a)	$I_n = \int_0^{\frac{\pi}{2}} \sin x . \sin^{n-1} x \, dx$	
	$= \left[-\cos x \sin^{n-1} x \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x . (n-1) \sin^{n-2} x \cos x \, dx$	M1 A1
	$= 0 + \ldots$	A1
	$= (n-1) \int_0^{\frac{\pi}{2}} \sin^{n-2} x \left(1 - \sin^2 x\right) dx$	
	$= (n-1) I_{n-2} - (n-1) I_n$	M1
	Leading to $I_n = \frac{n-1}{n} I_{n-2}$ (✱)	A1 (5)
(b)	$\int_0^{\frac{\pi}{2}} x\left(\sin^5 x \cos x\right) dx = \left[\frac{x \sin^6 x}{6} \right]_0^{\frac{\pi}{2}} - \frac{1}{6} \int_0^{\frac{\pi}{2}} \sin^6 x \, dx$	M1 A1
	$I_6 = \frac{5}{6} \times \frac{3}{4} \times \frac{1}{2} I_0 = \frac{5}{6} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2} \left(= \frac{5\pi}{32} \right)$	M1 A1
	Hence $\int_0^{\frac{\pi}{2}} x\left(\sin^5 x \cos x\right) dx = \frac{\pi}{12} - \frac{1}{6} \times \frac{5\pi}{32} = \frac{11\pi}{192}$	A1 (5)
		(10 marks)

Question number	Scheme	Marks
6. (a)	$\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 4 \\ -4 \end{pmatrix}, \quad \overrightarrow{QR} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$	B1
	$\overrightarrow{PQ} \times \overrightarrow{QR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 4 & -4 \\ 2 & 2 & -2 \end{vmatrix} = \begin{pmatrix} 0 \\ -14 \\ -14 \end{pmatrix}$	M1 A2, 1, 0 (4)
(b)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = -2 \quad \Rightarrow \quad y + z = -2$ or equivalent	M1 A1 (2)
(c)	$y + z = -2$	
	$x + y - z = 6$	
	Let $z = \lambda \quad \Rightarrow y = -\lambda - 2, x = 2\lambda + 8$	M1 A1 A1
	$\mathbf{r} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$	M1
	$\left(\mathbf{r} - \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} \right) \times \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \mathbf{0}$	A1 (5)
		(11 marks)

Question number	Scheme	Marks
7. (a)	$\begin{pmatrix} 2 & k & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 3k+18 \\ 12 \\ -8 \end{pmatrix} = \lambda\begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix}$	M1
	$\lambda = 4 \quad \Rightarrow \quad 3k+18=36 \quad \Rightarrow \quad k=6$	M1 A1 (3)
(b)	$\lambda = 4$ is an eigenvalue	B1
	$\begin{vmatrix} 2-\lambda & 6 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & -2 & 1-\lambda \end{vmatrix} = (1-\lambda)\left((2-\lambda)(1-\lambda)-6\right)$	M1
	$(\lambda-1)\left(\lambda^2-3\lambda-4\right) = (\lambda-1)(\lambda-4)(\lambda+1)$	M1
	$\lambda = (4,)\ 1,\ -1$ 1 and -1	A1 (4)
(c)	$\begin{pmatrix} 2 & 6 & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} t-2 \\ t \\ 2t \end{pmatrix} = \begin{pmatrix} 8t-4 \\ 2t-2 \\ 0 \end{pmatrix}$	M1 A2,1,0
	$x = 8t-4,\ y = 2t-2,\ z = 0$	
	$x - 4y - 4 = 0$	M1 A1 (5)
		(12 marks)

Question number	Scheme	Marks
8. (a)	$\frac{dx}{du} = 5\sec u \tan u,\ \frac{dy}{du} = 3\sec^2 u$	
	$\frac{dy}{dx} = \frac{3\sec^2 u}{5\sec u \tan u} \quad \left(= \frac{3}{5\sin u}\right)$	B1
	$y - 3\tan u = \frac{3}{5\sin u}(x - 5\sec u)$	M1
	Leading to $3x = 5y\sin u + 15(\sec u - \tan u \sin u)$	
	$3x = 5y\sin u + 15\left(\frac{1-\sin^2 u}{\cos u}\right)$	
	$3x = 5y\sin u + 15\cos u \quad$ (✱)	M1
		A1 cso (4)
(b)	Equations of asymptotes $\quad y = \pm\frac{3}{5}x \quad$ both	B1
	Eliminating y or x between $3x = 5y\sin u + 15\cos u$ and $y = \frac{3}{5}x$	
	$3x = 3x\sin u + 15\cos u$	
	$x = \frac{5\cos u}{1-\sin u},\quad y = \frac{3\cos u}{1-\sin u}$	M1 A1
	Similarly between $3x = 5y\sin u + 15\cos u$ and $y = -\frac{3}{5}x$	
	$x = \frac{5\cos u}{1+\sin u},\quad y = -\frac{3\cos u}{1+\sin u}$	M1 A1
	Let (x_M, y_M) be the coordinates of the mid-point of RS.	
	$x_M = \frac{1}{2}\left(\frac{5\cos u}{1-\sin u} + \frac{5\cos u}{1+\sin u}\right) = \frac{5\cos u}{2}\left(\frac{2}{1-\sin^2 u}\right) = 5\sec u$	M1
	$y_M = \frac{1}{2}\left(\frac{3\cos u}{1-\sin u} - \frac{3\cos u}{1+\sin u}\right) = \frac{3\cos u}{2}\left(\frac{2\sin u}{1-\sin^2 u}\right) = 3\tan u$	A1
	The coordinates (x_M, y_M) are the same as P.	
	P is the mid-point of RS. (✱)	A1 cso (8)
		(12 marks)